An Adaptive Stochastic Model for Financial Markets

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Abstract

An adaptive stochastic model is introduced to simulate the behavior of real asset markets. The model adapts itself by changing its parameters automatically on the basis of the recent historical data. The basic idea underlying the model is that a random variable uniformly distributed within an interval with variable extremes can replicate the histograms of asset returns. These extremes are calculated according to the arrival of new market information. This adaptive model is applied to the daily returns of three well-known indices: Ibex35, Dow Jones and Nikkei, for three complete years. The model reproduces the histograms of the studied indices as well as their autocorrelation structures. It produces the same fat tails and the same power laws, with exactly the same exponents, as in the real indices. In addition, the model shows a great adaptation capability, anticipating the volatility evolution and showing the same volatility clusters observed in the assets. This approach provides a novel way to model asset markets with internal dynamics which changes quickly with time, making it impossible to define a fixed model to fit the empirical observations.

Keywords:
Stochastic processes, financial markets, probability distributions, stylized facts, fat tails, volatility clusters, power laws.

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1. Introduction

A huge number of works aimed at describing the financial markets dynamics have been presented over the last decades. Although many models have been proposed, using different approaches, they usually can only reproduce a subsample of the characteristics observed in real market. The complexity of markets dynamics has led the scientific community to divide the problem into several lines of research, such as the study of the probability distribution of the asset returns or the mechanisms why such returns show certain autocorrelations. Hence, many aspects of financial markets remain undiscovered and new models should be proposed in order to complete its understanding.

From a statistical point of view, several probability distributions have been introduced as candidates to fit the asset returns histograms of financial markets: Lévy [1], Normal Inverse Gaussian [2] and KR [3] distributions among others. Besides that, other probability distributions have been proposed for internal processes related to price formation which might induce the heavy tailed distributions of asset returns. Such is the case of the Student distribution assigned to the order placement process in the order book [4, 5]. An important problem arises when trying a theoretical probability distribution to fit with an asset returns histogram, since the latter usually shows fat tails allocating excessive probability to large returns. This mismatch with the theoretical tails led to one of the most active lines of research: the search for more suitable distributions and the underlying mechanisms which induce those empirical large tails [6, 7]. In this direction an important conclusion is that the empirical histograms seem to be described by power laws with very similar exponents for different types of markets [8, 9].

On the other hand there are additional facts which can not be explained from a statistical point of view. While the Autocorrelation Function of returns decays rapidly with time, the Autocorrelation Function of absolute returns remains significant indicating positive autocorrelation [7], which has been related to another well-known effect: volatility clustering [7, 10]. It consists in bursts of high volatility located within precise time intervals, diverging considerably from the average behavior. Volatility clusters are a typical characteristic of financial markets.

Many models, with different approaches, have been proposed with the aim of reproducing some of the previous characteristics in order to identify the basic precursor mechanisms for them. Next, a general review of the most important kinds of models is shown.

A relevant group of models is the so-called "Agent models", which attempt to explain a financial market as an aggregation of single agents, or clusters of agents, with specific roles assigned. Every role would be a basic investment behavior. A good overview of agent models can be found in [11]. While belonging to the same group, a variety of different models have been introduced, depending on the roles assigned to the involved agents [12, 13, 14]. Some of these works are focused on how an agent-based model can reproduce some real markets characteristics [15, 16]. It is worth mentioning the so-called "herd behavior" i.e. of the appearance of imitation patterns among the investors which might be responsible for the observed autocorrelation in some periods of an asset evolution [17, 18, 19]. Although herd behavior is usually related to agent-based models other types of models may incorporate this concept.

The order-book models form another active line of research. The order-book, in a rough view, is a computing system which gathers all the incoming sell and buy limit orders, placing them along a price axis according to their price targets. Along with these limit orders, market and cancellation orders are already introduced in the book. The former consist in orders to be matched with the best price at the moment, while the latter are cancellations of previous limit orders. The interactions among all these orders constitute the basic dynamics of price formation. Thereby the models of the order-book aim to capture the essence of how the price of an asset is built [4, 20, 21, 22].

Modeling by means of stochastic processes is a natural approach, since the asset returns evolution seems to be a random result of the combination of the decisions of many thousands of investors. Several works have introduced stochastic models [23, 24]. As commented previously, some phenomena related to autocorrelation have been found in financial markets. Hence, a pure stochastic process without inducing any degree of autocorrelation would not be realistic enough. So, these stochastic models are usually tested on whether they reproduce those phenomena of the real markets.

In addition to those models grouped together in the previous paragraphs, other approaches have been proposed such as the "Microscopic spin model" [25] or models based on the "Quantum field theory"[26], among others.

This work introduces an adaptive stochastic model which can change its parameters endogenously by using the recent historical data, in order to assimilate the new information incorporated in an asset market. The basic idea underlying the model is that the multiplication of two different uniform probability distributions can replicate any empirical histogram when the parameters of both distributions are fitted appropriately. The idea of this multiplication of two uniform distributions resulted from a general Dynamics of Resource Density which is also presented in this work.

In one sense, the Dynamics of Resource Density is a macroscopic view of the order-book state over a time interval, after aggregating all the single orders received along such an interval. However, it is not a microscopic view of how the processes interact in the order-book, but a sequence of macroscopic equivalent states resulted from the combination of the microscopic processes. So, the Dynamics of Resource Density is not an order-book model, but a high level model of the evolution of supply and demand.

The first goal of this work is to demonstrate that a general simulator of probability distributions is derived from a generic Dynamics of Resource Density, consisting in the multiplication of two uniform distributions with parameters depending on the specific system under study. The second goal is to build an adaptive stochastic process with the previous simulator, fitting automatically its parameters according to the historical evolution of the asset.

This model is applied in the paper to three well-known indices: the Spanish Ibex35, the American Dow Jones and the Japanese Nikkei, thereby covering European, American and Asian markets. The time series used are the daily returns of every index from 2008 to 2010. It is found that the histograms produced by the model and the real ones are really close, showing the same fat tails and power laws. Moreover, the model replicates correctly the real autocorrelation structure and volatility clustering.

The work is organized as follows. The Dynamics of Resource Density is presented in Section 2, introducing the dynamics itself and obtaining a general probability simulator, based on the multiplication of two uniform distributions. In the same section it is demonstrated that the previous multiplication is equivalent to only one uniform distribution defined in an interval with random limits. The adaptive stochastic model is introduced in section 3. Applications of the model to Ibex35, Dow Jones and Nikkei indices are shown in section 4. Finally, the conclusions of the work are summarized in Section 5.

2. Dynamics of Resource Density

The dynamics of a distributed resource with border constraints, affected by an external demand, is studied in this section. This is a specific case of the most general situation in which a distributed resource evolves when external demands can take place in any part of its distribution. An example of the general case is a predator feeding on a specimen in an ecosystem. Here, the resource density is the distribution of the specimen within the ecosystem, that is a function of two or three dimensions, depending of the ecosystem in question. The external demand is the predator's need for feeding, inducing a resource consumption within the whole surface or volume of the ecosystem. In this example there is not any kind of constraint, since the resource can be consumed in any part of the ecosystem. On the other hand, the growth processes can be represented as resource densities with border constraints. For example, the tumoral growth in a patient is a process in which the tumoral cells, belonging to the border of the tumor, demand space in the surrounding healthy area in order to continue with the growing process. Thereby this process is in fact a competition between healthy and tumoral cells for space [27]. In this example the resource density is the volumetric density of surrounding space and the external demand is the need for space in the tumoral surface. It is clear that, in this case, there is a constraint since the resource (space) is only consumed in the border of the resource distribution.

It will be seen in subsequent sections that the evolution of the price of a financial asset can be described as the dynamics of a linear resource density with a border constraint, being the price of the asset the boundary of the density. In this section, a general study of the dynamics of a linear density with a border constraint is carried out without any reference to financial markets. The application of the obtained results to financial markets is done in 3.1.

2.1. General dynamics of a linear resource density with a border constraint

Let us consider a generic resource distributed along a one-dimensional variable x. This resource is distributed according to a linear density

$$\rho_t(x) = \frac{dm_t(x)}{dx} \tag{1}$$

which is a function of time, being $m_t(x)$ the quantity of resource allocated in x for time t . This density has a boundary in x_0 in such a way that $\rho_t(x) = 0$ if $x < x_0$ and $\rho_t(x) \neq 0$ if $x \geq x_0$. Let us consider now an incoming demand for this resource which starts consuming from the boundary x_0. Hence, as the

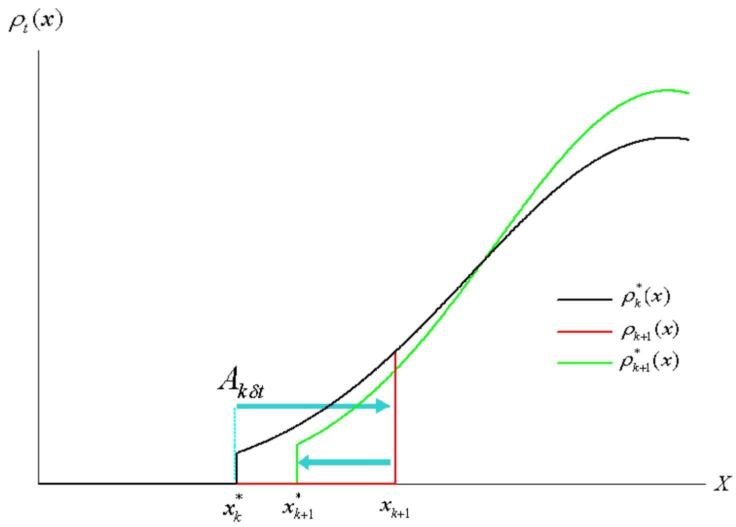

Figure 1: Resource density evolution. Basic process for the time interval $[k\delta t, (k+1)\delta t]$.

resource is consumed the boundary moves in the increasing direction of the variable x until all the demand has been satisfied, setting the new boundary in $x_1 > x_0$. Afterwards, the dynamics of $\rho_t(x)$ continues modifying the resource allocation along the axis while new quantities of demand come into the system. So, the evolution of the boundary x_t is resulted from the interaction between two processes: the dynamics of $\rho_t(x)$ as a generator of the distributed resource and the incoming demand which consumes that resource starting always from the current boundary x_t.

The evolution of the boundary within a short time interval $[k\delta t, (k+1)\delta t]$ is described by the next "Resource Density Law":

$$A_k = \int_{x_k^*}^{x_{k+1}} \rho_k^*(x)dx \tag{2}$$

where $\rho_k^*(x)$ and x_k^* are respectively the density and the boundary just at the beginning of the interval, while A_k is the incoming demand within the same interval. So, the unknown quantity is x_{k+1} which is the position of the boundary once the incoming demand A_k has interacted with the initial conditions ($\rho_k^*(x)$,x_k^*).

When applying eq. 2 to successive time intervals a discrete time evolution of the boundary: $x_t = x_{\delta t}, \ldots, x_{N\delta t}$ is obtained. The process can be visualized as a series of discrete steps:

1. The density dynamics originates $\rho_k^*(x)$ at $t = k\delta t$ with boundary in x_k^*.
2. The density dynamics remains "frozen" within δt, while the incoming demand along the time interval consumes resource in the increasing direction of the axis, beginning in x_k^* and ending in x_{k+1}.
3. The density dynamics acts again making the distribution evolve until reaching a new state $\rho_{k+1}^*(x)$, moving the boundary from x_{k+1} to x_{k+1}^*.

The process is shown in Fig. 1. Notice that the resource generation and the resource consumption do not take place at the same time, but the former acts at the beginning of the time interval whereas the latter does it along the whole interval. A second consideration is that when density dynamics acts again, at the beginning of the next time interval, the boundary is shifted from x_{k+1} to x_{k+1}^*, which means that the final boundary in $[k\delta t, (k+1)\delta t]$ can not be taken as the initial boundary in $[(k+1)\delta t, (k+2)\delta t]$. Although in a real process both, generation and consumption take place at the same time, it will be shown in next sections that this simplification of short time intervals with all the generation concentrated at the beginning of the interval leads to results compatible with those of the real systems.

2.2. A special case. Constant density.

Once the processes $\rho_t^*(x)$ and A_t, which are specific of the system under consideration, are known, the evolution of the boundary x_t can be described by Eq. 2. In order to define these processes, let us consider the evolution of the boundary $x_t = x_1, x_2, \ldots, x_N$, along the time periods $[0, \delta t], [\delta t, 2\delta t], \ldots, [(N-1)\delta t, N\delta t]$. The resource densities involved in the whole period $[0, N\delta t]$ are

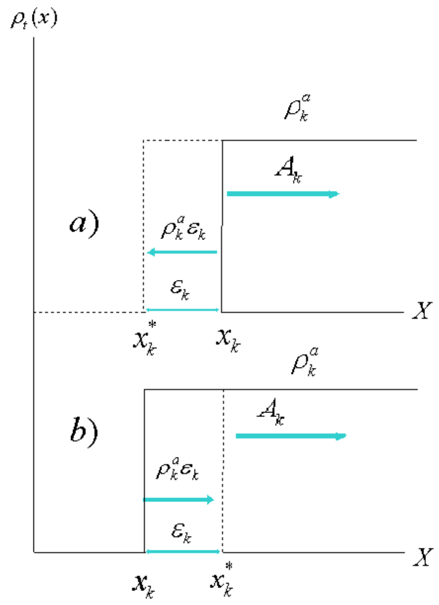

Figure 2: Scheme of the external and internal demands in a dynamics of a linear resource density. a) Working in opposite directions. b)Working in the same direction.

$\rho_t = \rho_0^*(x), \rho_1^*(x), \ldots, \rho_{N-1}^*(x)$, which are the ones formed just at the beginning of every single period. According to Eq. 2 only the density area between x_k^* and x_{k+1} plays a role. Let us consider now an averaged constant density ρ_0^a gathering the effect of all the single density pieces involved in the time period $[0, N\delta t]$. By applying eq. 2 to this averaged density within the interval $[0, N\delta t]$ a handy expression is obtained

$$x_N - x_0^* = \frac{A_0}{\rho_0^a} \tag{3}$$

where x_N is the boundary at the end of the period $[0, N\delta t]$, x_0^* the boundary just at the beginning of the same period, A_0 the incoming demand for resources and ρ_0^a the averaged density which summarize the effect of the real density dynamics of the process. Notice, as already commented for the general dynamics of single periods, that x_N is the boundary at the end of its period, but is not the one at the beginning of the next period, since a new density ρ_N^* will be reconfigured with a new boundary x_N^*.

The generalization of eq. 3 for an arbitrary time step τ

$$x_{k+1} - x_k^* = \frac{A_k}{\rho_k^a} \tag{4}$$

is applicable over the "macroscopic" time interval $[k\tau, (k+1)\tau]$.

From eq. 4 follows

$$x_{k+1} - (x_k + \epsilon_k) = \frac{A_k}{\rho_k^a} \tag{5}$$

where ϵ_k is the evolution of the boundary from x_k to x_k^* in $t = k\tau$. Let us call ϵ_k "Border effect". Notice that ϵ_k can be positive or negative, since the resource distribution may reorganize itself moving to either the positive or the negative directions of the axis.

Once eq. 5 is reorganized the final law is

$$\Delta x_k = \frac{\hat{A}_k}{\rho_k^a} \tag{6}$$

being $\Delta x_k = x_{k+1} - x_k$ and $\hat{A}_k = A_k + \rho_k^a \epsilon_k$.

Let us call $\hat{A}_k$ "Compound Demand" since it is equivalent to a mixed demand for resource, consisting of A_k and $\rho_k^a \epsilon_k$, which are the external and internal demands respectively. The latter is fictitious and generated internally by the resource dynamics since both, ρ_k^a and ϵ_k, are parameters related to the evolution of the resource distribution. $\rho_k^a \epsilon_k$ corresponds to an area in the distribution equivalent to the external demand needed to move the boundary from x_k to x_k^*. Notice that this quantity may be positive or negative, acting for or against the external demand, respectively.

Fig. 2 shows that $\rho_k^a \epsilon_k$ can be considered as an additional demand in a scenario with a static resource distribution. This quantity can be either positive or negative, contrary to A_k which is always positive, thereby external and internal demands can add or counteract their effects. This idea, of translating the internal dynamics of the resource distribution into an equivalent external demand for the distribution itself, may seem a little bit strange, but it allows to reduce a system with an internal dynamics affected by an external force to two different forces acting against a static resource distribution.

2.3. A generic simulator of probability distributions.

Although eq. 6 provides a simple and clear relation among the variables involved in the evolution of a resource distribution interacting with an external demand, such relation is still more qualitative than quantitative.
For practical purposes both $\hat{A}_k$ and $1/\rho_k^a$ are proposed to be random variables distributed according to an uniform probability distribution, defined within the intervals $[\hat{A}^m, \hat{A}^M]$ and $[1/\rho^M, 1/\rho^m]$ respectively, where "m" indicates the minimum possible value of the variable and "M" the maximum one. According to this definition, Δx_k becomes a random variable in eq. 6, resulted from the multiplication of two random variables uniformly distributed where $\hat{A}_k \in [\hat{A}^m, \hat{A}^M]$ and $1/\rho_k^a \in [1/\rho^M, 1/\rho^m]$.

It will be shown in next sections that the multiplication of two random variables uniformly distributed is a generic simulator of empirical histograms, when the limits of the intervals are fitted appropriately. Notice that this multiplication of two uniform variables with static intervals is equivalent to an unique variable distributed uniformly within an interval with random extremes. Once the variable $\hat{A}_k$ takes a value, let us say ζ, the variable Δx_k will be distributed uniformly within the interval $[\zeta/\rho^M, \zeta/\rho^m]$. On the other hand, if $1/\rho_k^a$ takes the value φ the variable Δx_k will be distributed uniformly within $[\varphi\hat{A}^m, \varphi\hat{A}^M]$. Hence, eq. 6 becomes the following expression

$$\Delta x_k = \theta_k \tag{7}$$

where $\theta_k \in [\hat{A}_k/\rho^M, \hat{A}_k/\rho^m]$ or $[\hat{A}^m/\rho_k^a, \hat{A}^M/\rho_k^a]$ which are equivalent intervals with random extremes.

3. Adaptive Sthochastic Model

3.1. Dynamics of resource density applied to asset markets.

The equivalence between the dynamics of a resource density and the one of an asset market is really intuitive: the resource density $\rho_k(x)$ is the available volume of the asset distributed within a logarithmic price axis x, according to the sellers expectations, and the incoming demand A_k is the volume of buyers wanting to acquire an asset at the current price. In other words, while $\rho_k(x)$ is the distribution of the supply of an asset along the price axis A_k is the demand of that asset at the current market price. Hence, the left boundary of the density is the current price since the flow of buyers always buy at the cheapest available price. This is why the incoming demand A_k always starts consuming the resource from the boundary, starting at the best offered price and continuing in the increasing direction of the price axis (border constraint).

According to the study carried out in section 2, the form of the real asset density ρ_k is substituted with an average density ρ_k^a. The combination of such average density with the demand A_k is finally described as a random variable θ_k in eq.7. According to this equation, and taking into account that Δx is the return in a logarithmic price axis, the return of an asset would be described by a random variable uniformly distributed within an interval with variable extremes.

3.2. Relationship between the resource density and the real densities in the order book.

The real price formation takes place in the order book as a result of the interaction between two different densities, the buyer and seller ones. The dynamics in the boundary between both densities determines the price. Many efforts have been done to find out the real form of those densities and their dynamics [4, 5]. While the model proposed in this work only takes into account one density, the seller one $\rho_k(x)$, the incoming demand A_k can be considered as the part of the buyer density which plays a role in k. So in a sense, the model is also considering two densities, although only the one belonging to the resource (asset) distribution is visualized as a density. Hence, the model introduces a model of supply and demand instead of a model of order book.

In any case, the model does not try to set the real form of the asset distribution, since the part of such distribution which is involved in the price formation in k is combined with the current demand in order to be substituted with a random variable, according to eq.7. Thereby, the model does not manage real densities, but it measures the effect of the interaction between them.

3.3. An adaptive model.

The evolution of an asset return in terms of a stochastic process of uniform random variables defined within an interval with variable extremes can be described by eq.7. These extremes depend on specific parameters of the system under study: the maximum and minimum values of $\hat{A}_k$ and ρ_k^a. The approach to be followed in this work is the calculation of the interval extremes by using the recent historical data of the asset evolution, instead of trying to evaluate the internal parameters of the system.

Let us consider the evolution of an asset return $r_k = r_1, r_2, \ldots, r_N$ and its moving average $m^h(r_k)$, calculated with the last h returns $\{r_{k-(h-1)}, \ldots, r_k\}$. The interval in which is defined every random variable θ_k from eq.7 is

$$\theta_k \in [m^h(r_k) - \gamma\hat{\sigma}_k, m^h(r_k) + \gamma\hat{\sigma}_k] \tag{8}$$

where $\hat{\sigma}_k$ is the estimate of the asset volatility in k, and γ is a scale factor. The volatility is estimated as follows

$$\hat{\sigma}_k = \sigma_k^L e^{\hat{r}_k^\sigma} \tag{9}$$

where σ_k^L is the typical deviation in k, calculated with the last L values of the asset returns $\{r_{k-(L-1)}, \ldots, r_k\}$, and $\hat{r}_k^\sigma$ is the averaged volatility return in k:

$$\hat{r}_k^\sigma = m^h(r_k^\sigma) \tag{10}$$

being $m^h(r_k^\sigma)$ the moving average of the volatility returns, where the volatility has been calculated with the last L values of the asset returns.

Finally, the evolution of the asset returns is described by the following stochastic process

$$r_k = \theta_k \tag{11}$$

where θ_k is a set of random variables uniformly distributed in the intervals given in (8).

Notice that r_k was defined as $lnx_{k+1} - lnx_k$, so the interval in $t = k$ is built with the data available in $t = k$ and determines the simulation of the price in $t = k + 1$.

4. Application to real markets

In this section the adaptive stochastic process defined by eqs. 11 and 8, with parameters γ, h and L, is applied to three well-known indices: the American Dow Jones, the Japanese Nikkei and the Spanish Ibex35. The historical data to be tested correspond to the daily returns for three complete years: 2008-2010.

The parameters of the model, L and h, define the amount of historical values to be taken into account, and γ is a scale parameter. According to this, the same model with identical parameters, builds different intervals of definition since the historical data of each asset are completely different.

According to eq. 11, the interval of definition of θ_k is built with the available information in $t = k$ in order to simulate the next return $r_k = lnx_{k+1} - lnx_k$. Once the interval has been fixed, the uniform random variable θ_k can generate as many executions of the model as needed to be compared with the real value of the asset in $k + 1$.

4.1. Adjustment of the parameters of the model and distribution of returns

The distribution of returns belonging to the real indices are compared with those produced by the model by using the Shannon entropy or information [28]. The possible values of the variable return r_k are divided into N subdivisions in order to obtain a discrete variable with values $\{x_1, \ldots, x_N\}$. The information of the event $r_k = x_i$ is

$$H(x_i) = -log_2(p(x_i)) \tag{12}$$

where $p(x_i)$ is the probability of $r_k = x_i$.

The average entropy, H, is given by

$$H = -\sum_{i=1}^{N} p(x_i) log_2(p(x_i)) \tag{13}$$

In order to get a measure of the entropy of the index we computed H for the real returns r_k of the index by using eq.13. On the other hand H^j_{model} correspond to the computation of H for a single execution of the process given by eq.11. Finally, H_{model} is calculated by averaging H^j_{model} over thousands of process executions

$$H_{model} = \frac{\sum_{j=1}^{M} H^j_{model}}{M} \tag{14}$$

where M is the number of model executions.

The absolute error of the averaged entropy $\epsilon = |H_{index} - H_{model}|$ provides a measure of how close both distributions are.

The values of the parameters of the model are adjusted to minimize this error, using a partition with $N = 60$ in eq.13 and setting $M = 2000$ in eq.14. We have found that the parameter which leads to larger variations in the absolute

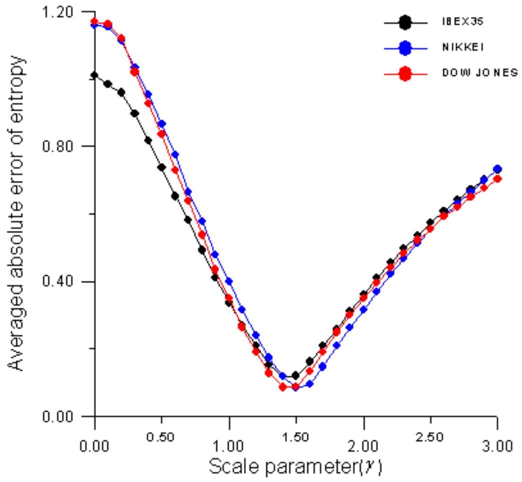

Figure 3: Absolute error of the Shannon entropy as a function of the parameter γ.

error is γ, hence we proceed to first adjust this scale parameter, letting h and L take values randomly within the intervals $[1, 10]$ and $[2, 10]$ respectively.

The absolute error as a function of γ is shown in fig.3. It can be seen that there is a clear minimum in 1.5 for all the indices. These minimum in γ is found independently from the values of h and L since they have been randomly chosen for each single model execution. Therefore, the scale parameter is set to $\gamma = 1.5$ for all the indices.

Once adjusted the scale parameter we proceed to adjust the parameter h, letting L take again random values within $[2, 10]$. The fig.4 shows that there are minima in $h = 7$, $h = 7$ and $h = 5$ for Ibex35, Dow Jones and Nikkei respectively. So, h is set to the corresponding minimum for each index from now forth.

Finally, the adjustment of the parameter L is carried out by calculating the absolute error as function of L, once the previous parameters γ and h have been set for each index. It can be seen in fig.5 that the optimal values are reached in $L = 5$, $L = 4$ and $L = 5$ for Ibex35, Dow Jones and Nikkei respectively.

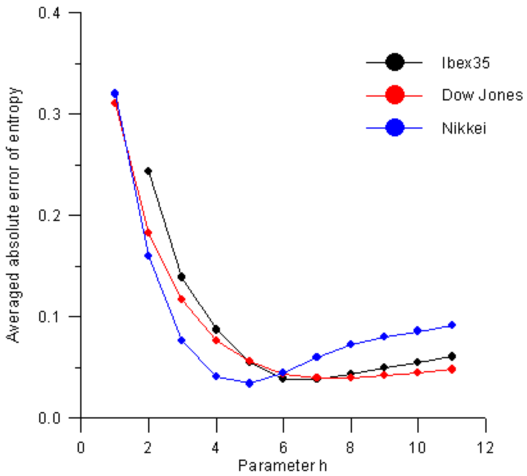

Figure 4: Absolute error of the Shannon entropy as a function of the parameter h.

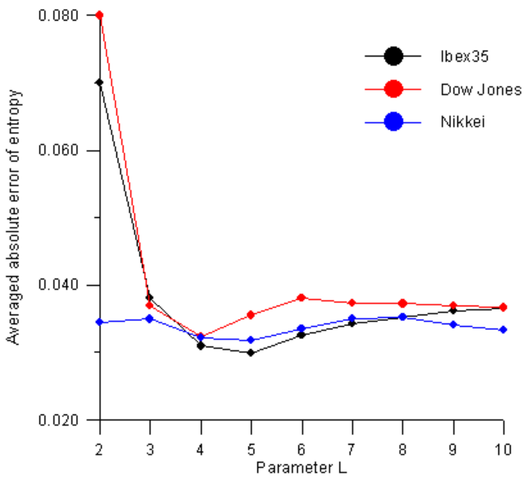

Figure 5: Absolute error of the Shannon entropy as a function of the parameter L.

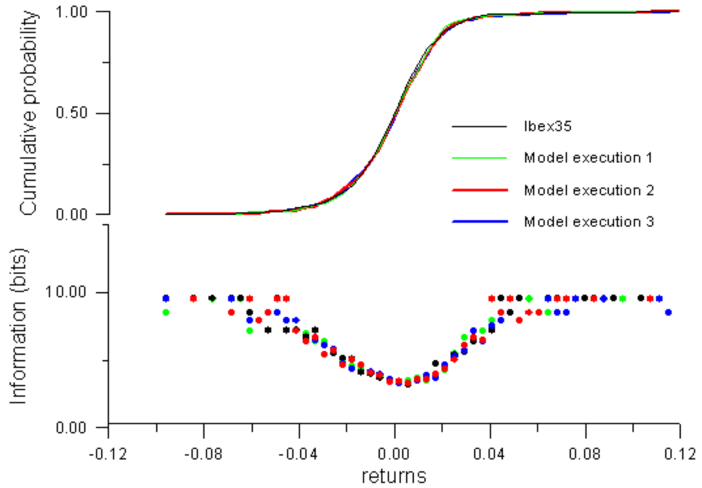

Figure 6: Comparison of cumulative probability distributions of returns (top) and Information (bottom) for Ibex35 and three executions of the corresponding adaptive model.

Hence, the final models are Ibex35 ($\gamma = 1.5, h = 7, L = 5$), Dow Jones ($\gamma = 1.5, h = 7, L = 4$) and Nikkei ($\gamma = 1.5, h = 5, L = 5$). However, notice that an unique model could have been chosen for the whole indices with values ($\gamma = 1.5, h = 6, L = 4$), since these values produce the same absolute error in all the cases and are set very close to the minimum of every single index. In any case, from now forth, all the simulations of the models are carried out with the values which optimize every single asset.

Now the probability distributions of returns produced by the models, with their parameters adjusted according to the previous paragraphs, are compared with those produced by the real indices. The top panel in figures 6, 7 and 8 shows the cumulative distributions of the index and three model executions. The bottom one shows the information $H(x_i)$ (eq.12) for the index and the model executions. It can be seen that the executions of the models fit accurately both, the cumulative probability and the information distributions.

The absolute error of the averaged entropy $\epsilon = |H_{index} - H_{model}|$ indicates

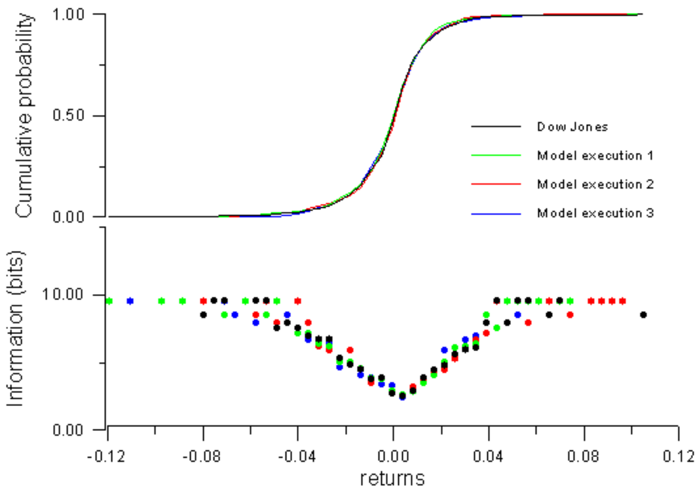

Figure 7: The same as fig.6 for index Dow Jones

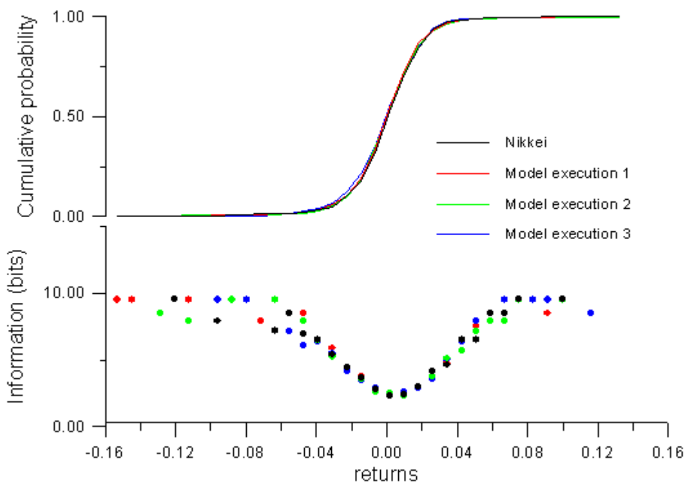

Figure 8: The same as fig.6 for index Nikkei

how close both distributions are. This error is very small, 0.03 for all cases, indicating that the model produces distributions really close to the real ones.

4.2. Power laws

In order to find out whether the probability of absolute returns can be written as a power law, $p(|r|) = |r|^{\alpha}$, it has been depicted $ln(p(|r|))$ as a function of $ln(|r|)$ in figure 9, for the indices under study and their corresponding adaptive models. It can be seen that real markets show power laws of their absolute returns and that their models fit very well those power laws, with exponents $-2.2, -2.2$ and -2.1 for Ibex35, Dow Jones and Nikkei respectively.

These results are in good agreement with previous studies which have already found power laws in financial markets [8, 9].

4.3. Fat tails

As already commented in Section 1, an important characteristic of the returns histograms is that they show fat tails, which indicates that large returns have a considerable probability of appearance. The search for the underlying mechanisms which induce those empirical large tails is an active line of research [6, 7].

In order to visualize the fat tails, the cumulative probabilities of the indices and their adaptive models are depicted as function of their absolute normalized returns in fig.10. The absolute normalized return is $r_k^* = |r_k - r^m|/\sigma$, being σ the standard deviation of r_k and r^m its mean value. The cumulative probability of the standard normal distribution is added to the graphic to show how far the index and model tails are from those of the normal distribution.
A detail of the tails is shown in the insets of the figure, confirming that the indices tails are much bigger than the ones of a normal distribution. On the other hand, the graphic shows that the models generate the same fat tails as the real indices.

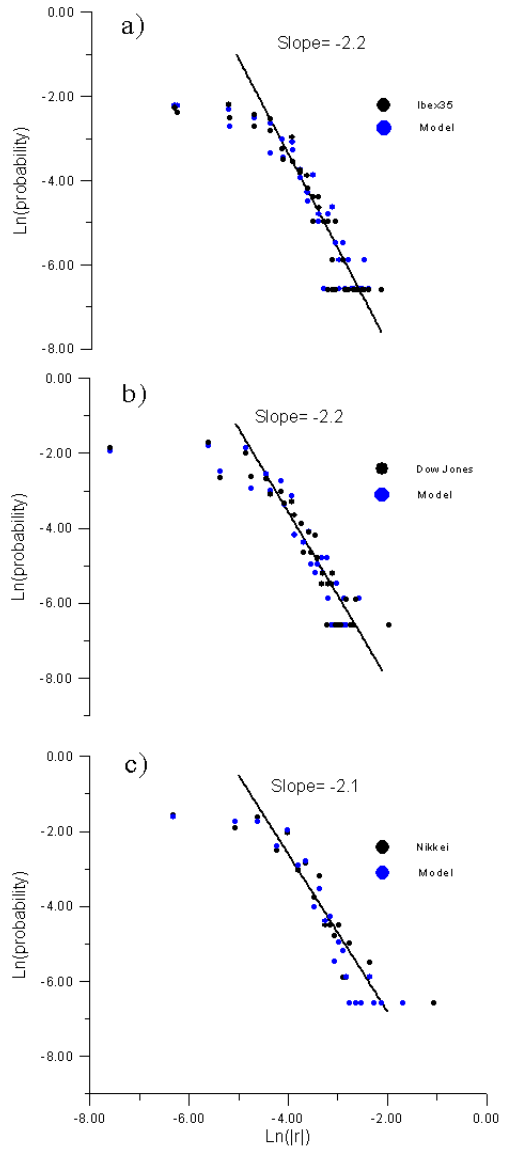

Figure 9: Power laws in the probability distributions of absolute returns. Comparison of the results of the indices (black dots): Ibex35 (a), Dow Jones (b) and Nikkei (c) with those of their corresponding adaptive models (blue dots).

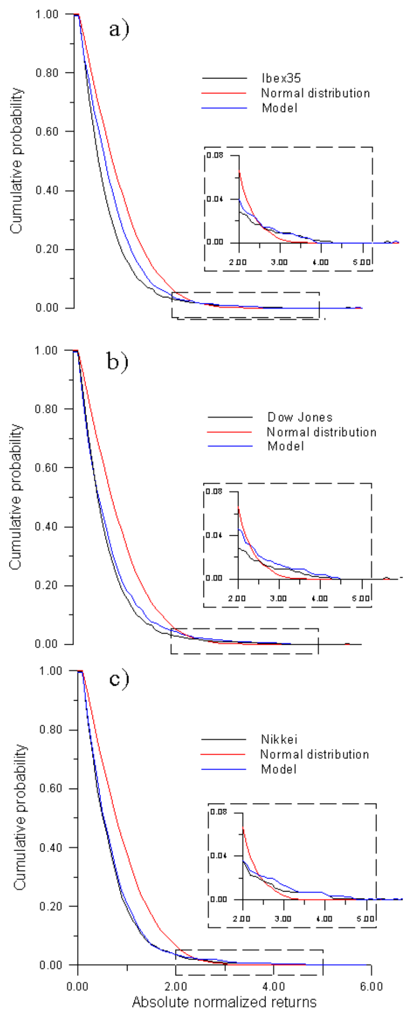

Figure 10: Fat tails in the cumulative probability distributions of absolute normalized returns. Comparison of the fat tails in Ibex35 (a), Dow Jones (b) and Nikkei (c) with those of their corresponding adaptive models and standard normal distribution.

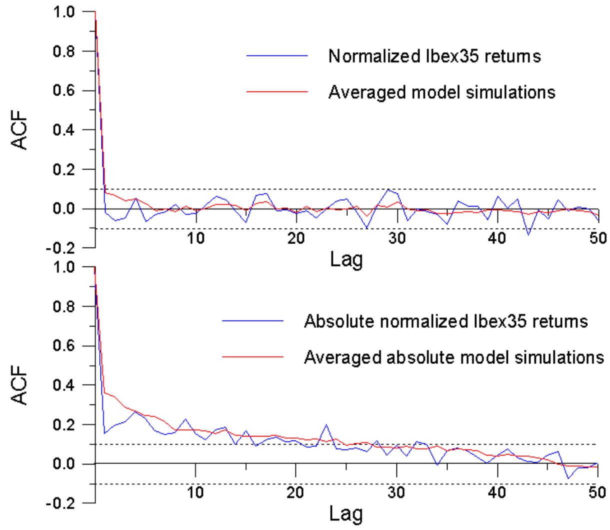

Figure 11: Autocorrelation function of Ibex35 (blue line) and the simulation results of its adaptive model (red line). Normalized returns (top). Absolute normalized returns (bottom).

4.4. Autocorrelation

Some of the markets characteristics can not be explained only from a statistical point of view. As commented in section 1, certain degree of autocorrelation has been found for returns. While the autocorrelation function of returns decays rapidly with time, the autocorrelation function of absolute returns remains significant indicating positive autocorrelation [7].

In Figures 11, 12 and 13 the autocorrelation functions (ACF) of each index and the simulations of their corresponding adaptive models are presented, both for normalized and absolute normalized returns. In the case of the models, the ACF corresponds to an average over fifteen executions. It can be seen that the model shows the same autocorrelation patterns as the real index, that is to say, there is no autocorrelation for normalized returns, but the autocorrelation of absolute normalized returns keeps significant for a period of time.

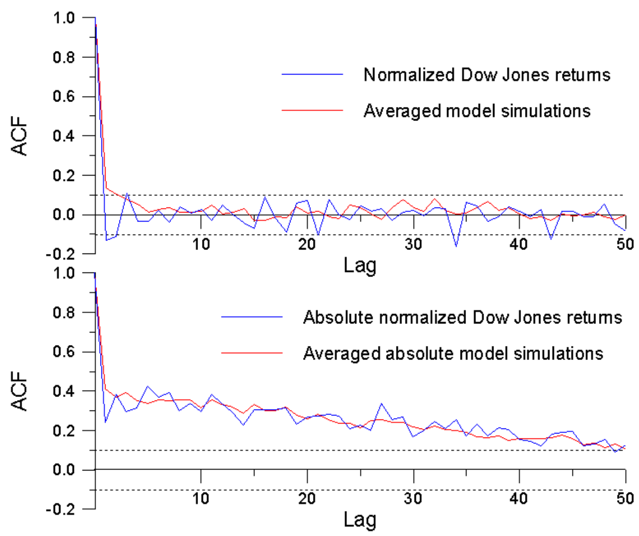

Figure 12: The same as fig.11 for Dow Jones.

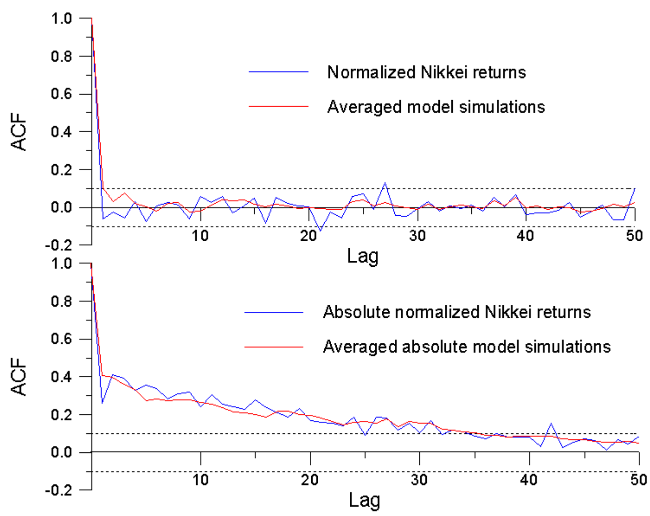

Figure 13: The same as fig.11 for Nikkei.

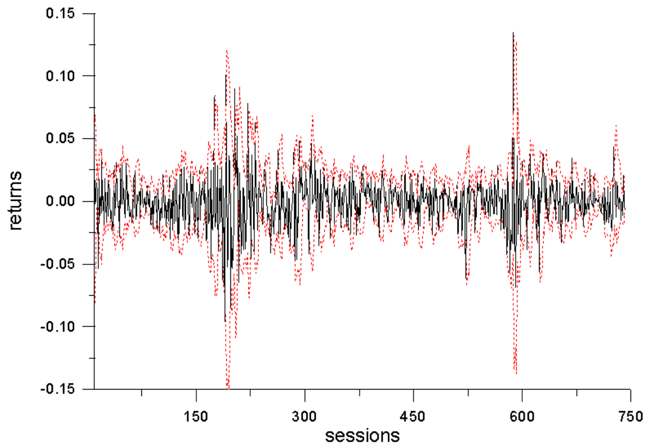

Figure 14: Volatility adaptation. Comparison of the time evolution of Ibex35 (black) with the time evolution of the extreme values in the corresponding adaptive model (red).

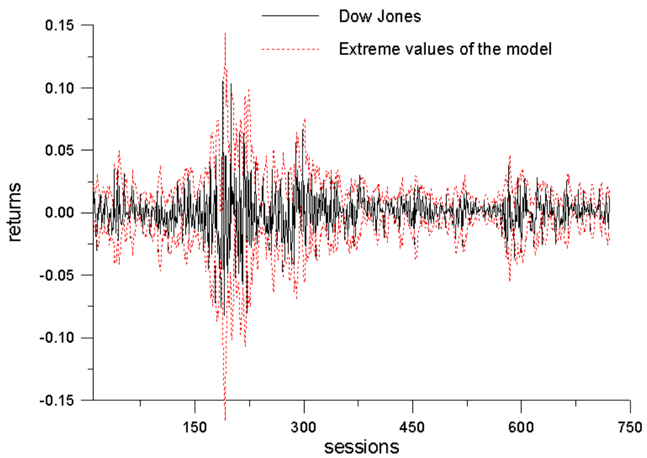

Figure 15: The same as fig.14 for Dow Jones.

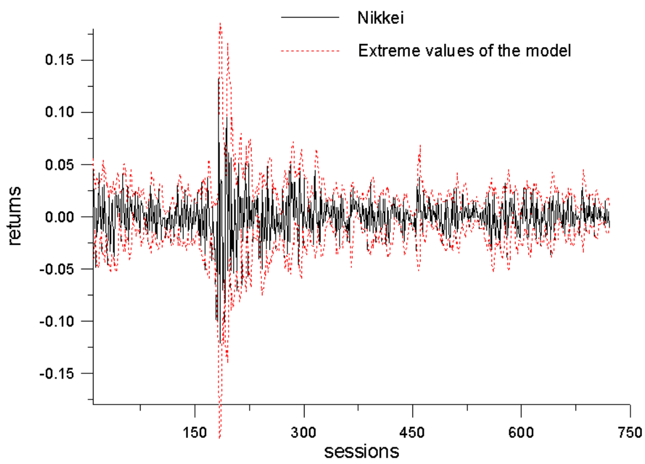

Figure 16: The same as fig.14 for Nikkei.

4.5. Adaptation capabilities of the model and volatility clusters

The evolution of the daily returns, as well as the extreme values of the model, along three complete years are shown in figures 14, 15 and 16 for Ibex35, Dow Jones and Nikkei respectively. A single execution of the model might be any return value between the extremes, with equal probability. The figures show that the model adapts quite well its parameters to follow the real evolution of the asset. The extreme values for certain value of t are the extremes of the interval in which the uniform variable is defined for that moment, so the adaptation of the extreme values in the figures is in fact the adaptation of the interval definition. On the other hand, it can be seen in the figures that this adaptation leads to reproduce the same volatility clusters observed in the real asset.

5. Conclusions

An adaptive stochastic model is introduced to simulate the behavior of real asset markets. It changes its parameters automatically by using the recent historical data of the asset. The basic idea underlying the model is that a random variable uniformly distributed within an interval with random extremes can replicate the histograms of asset returns. This basic idea is derived from a Dynamics of Resource Density, which is a model of the evolution of supply and demand, also presented in this work . The proposed stochastic process not only explains the empirical observations, but it also has an important theoretical support. The adaptive model is applied to daily returns of three well-known indices, Ibex35, Dow Jones and Nikkei, for three complete years (2008-2009).

The model can reproduce the histogram of the indices as well as their autocorrelation structures. Specially interesting is how the model produces the same fat tails as in the real asset histograms. It is also observed that the same power laws found in the indices are obtained with the model, having exactly the same exponents. On the other hand, the model shows a great adaptation capability, anticipating the volatility evolution and showing the same volatility clusters observed in the indices.

6. Acknowledgements

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